The developing structure of dynamical systems

H. P. Fang

Department of Physics, Fudan University, Shanghai 200433, China;*

Institute of Theoretical Physics, P.O. Box. 2735, Beijing, 100080, China.

Abstract

In order to investigate the evolutionary process of many deterministic dynamical systems with unfixed parameter, a set of dynamical models with parameter changing continuously and the accumulation of this change might be large is introduced and discussed. The boundary crises and the period-doubling bifurcations are found suppressed and scaling properties for these phenomena are exploited. Due to this suppression, the period-doubling bifurcations seem no longer continuous. We further discuss the possible applications of these models.

PACS number: 05.45. +b

Introduction. One of the practical methods to investigate the phenomena common in many scientific fields such as physics, biology, ecology, geophysics and economy, etc is by modeling them either with differential, difference-differential, or pure difference equations. After the models have been studied, these phenomena can be understood and even be predicted. In this Letter, we will introduce a model which can be used to model and then study the behavior of many natural systems with unfixed environment parameters.

The motivation of introducing such models is our recent carefully examination on the work which was tempted to apply the chaotic dynamics to understand and predict the natural phenomena [see, e.g. ref 1-5]. C. Nicolis and G. Nicolis proposed some evidence for the climate which caused a debate on whether climatic attractors exist or not[1]. The correlation dimensions[6] for many economical data sequences had also been calculated which were taken as the evidence that the economical chaos might exist [see, e.g., ref. 2]. And recently, the short term prediction algorithm was used to test the predictability of some economical data[3]. Chaos in heart and brain was also reported and tempted to be controlled[4]. All of these studies are based on the assumption that these natural systems are environment independence, or the data of these dynamical systems are take in a limited time interval so that accumulation of the environment parameter change is sufficiently small. Then the familiar concepts such as phase-space, bifurcation diagrams, orbits, attractors, return maps, basins of attractor, manifolds, etc. can be used to extract information of these natural systems [7-8]. In fact, many of the natural systems such as all biological, ecological and economical systems have developed into other ones before they reach any asymptotic states. The environment parameters change so quickly that accumulation of the parameter change is **significantly large** in a limited time interval which has changed the dynamical behavior of the systems. The data sequences from these systems, on one hand, are some kinds of transients in short term interval (so that the accumulation of the parameter change is small), and on the other hand, are the sets of points in different environment parameters in a relative long time interval. It is clear that the above listed techniques are applicable only when the change of environment is negligible. Thus, this assumption not only restricts the study on the global behavior of these natural systems, but also might cause misunderstanding.

In order to study the dynamical behavior of these natural systems, and clarify the possibility and limitations to apply the familiar concept to study natural systems, in this Letter we will study the dynamical models with *parameter changing continuously* and *the accumulation of this change might be large*. Explicitly, we will study the models in the form

$$\vec{X}_{n+1} = F(\vec{X}_n, \vec{P}_n), \tag{1}$$

$$\vec{P}_{n+1} = G(\vec{P}_n) + \vec{P}_n. \tag{2}$$

where $\vec{X}_n$ and $\vec{P}_n$ represent one-, two- or high-dimensional variable and parameter vectors, respectively. Considering the continuous evolutionary process of the natural systems, $G(\vec{P}_n)$ is usually small (If Eq. (1) represents a difference form of differential equations with very small time step δt, $G(\vec{P}_n) \to 0$ as $\delta t \to 0$). One particular case for this model has already been discussed in detail which is the periodic forced driven systems. Some of the special cases of this model with the parameter close to the bifurcation points or $G(\vec{P}_n)$ is a noise had also received attention recently [9−11]. In the present Letter, we will concentrate on the situation with the parameters change unperiodically and ever irregularly, and the accumulation of the change of the parameters might be large (hereafter we call these systems the structure-variable systems, namely, SVS's) . We will discuss the behavior and possible applications for these models.

Due to the unperiodic change of the parameters, the dynamical systems will never approach asymptotic states. All the data we observed are transients. When the change of the parameters is not too fast ($G(\vec{P}_n)$ is sufficiently small), for a short observation time interval, the SVS represented by eqs. (1-2) might exhibit some behavior similar to those of Eq. 1 with fixed parameters $\vec{P}_n$. For $\vec{P}_n$ close to the parameters for bifurcations or boundary crises[12] of Eq. 1, the behavior of the SVS (1-2) is significantly different from that of system (1) with fixed parameters. The crises and period-doubling bifurcations are suppressed and scaling properties about these phenomena can be found. Due to this suppression the periodic orbits and period-doubling bifurcations are difficult to be observed directly.

Example For a detail idea of the behavior of these SVS's we take the following map

$$\begin{cases} x_{n+1} = f(\mu_n, x_n) = 1 - \mu_n x_n^2, \\ \mu_{n+1} = \mu_n + \delta\mu, \end{cases} \tag{3}$$

as an example. Fig. 1 shows a typical case for a trajectory of this map. This diagram shares some similarity with the bifurcation diagram of the Logistic map in the same parameter interval. Unlike that in the bifurcation diagram, no point is thrown away as transients. In fact, this diagram reflects the evolutionary process of the dynamical systems described by the Logistic map (we will call it the developing diagram hereafter), which corresponds to an idea model of the insects population on an island[13] with the environment of the island changes (The area might shrink because of the housegreen or other effects). The value of the parameter step $\delta\mu$ reflects the rapidity of evolutionary process of the dynamical system. It is clear that when the width of any periodic window is small or comparable with the value of $\delta\mu$, this periodic window disappears in the developing diagram.

A direct observation of the Fig. 1 is that the period-doubling bifurcation seems no longer a continuous one. Comparing to the bifurcation diagram, the parameter of period-doubling is shifted. Now we estimate the rapidity of the convergence of the period-doubling bifurcations and the shift with respect to $\delta\mu$ in these developing diagram.

Consider a period-doubling bifurcation occurs at μ_c in the bifurcation diagram and at $\mu_d(\delta\mu)$ in the developing diagram with a parameter step $\delta\mu$. μ is the controlling parameter. $\mu_d(\delta\mu) > \mu_c$. In fact, from the parameter $\mu = \mu_c$, the developing diagram goes alone the lose-stability periodic orbit until it reaches $\mu = \mu_d$, due to the slowing down of the convergence and the inertia.

Consider a period p orbit which should be a certain fixed point x^* of the following map

$$x_{n+p} = \underbrace{f \circ f \circ \cdots \circ f(\mu, x_n)}_{p \text{ times}}. \tag{4}$$

Let

$$x_n = x^* + \epsilon_n,$$

$|\epsilon_n|$ diminishes as $e^{-A|\mu - \mu_c|}$ for each iterate of the map (4) at parameter μ, and A is a positive constant[14]. Consider that we go forward k steps from $\mu = \mu_c$ of map (4) and reach $\mu_d = \mu_c + k\delta\mu$. Assume that the initial deviation of the period p orbit is $\epsilon^{(0)}$, and the final deviation is $\epsilon^{(k)}$,

$$|\epsilon^{(k)}| \propto e^{-A\sum_{i=1}^{k}|\mu_i - \mu_c|}|\epsilon^{(0)}| = e^{-A\sum_{i=1}^{k}i|\delta\mu|}|\epsilon^{(0)}| = e^{-Ak(k+1)|\delta\mu|/2}|\epsilon^{(0)}|.$$

For all small $|\delta\mu|$ we assume $|\epsilon^{(k)}/\epsilon^{(0)}|$ to be an approximate constant, so that $k(k+1)\delta\mu$ is approximately a constant. Reminding that $k\delta\mu = \mu_d - \mu_c$, we obtain that

$$|\mu_d(\delta\mu) - \mu_c| \propto \sqrt{|\delta\mu|}. \tag{5}$$

We have checked this relation numerically for many period-doubling bifurcations in many models. This relation holds approximately provided the parameter step $\delta\mu$ is not too large.

At $\mu = \mu_d$ the rapidity of convergence is proportional to

$$e^{-A|\mu_d - \mu_c|}.$$

The larger the value of $|\mu_d - \mu_c|$, the smaller the value of $e^{-A|\mu_d - \mu_c|}$. Thus the larger the value of $\delta\mu$, the more rapid the convergence (see Eq. 4). This makes the period-doubling bifurcation no longer a continuous bifurcation for not too small $\delta\mu$.

The parameter for a boundary crisis is also shifted as shown in Fig. 2. In a similar way, we get the scaling law for the parameter shift $\Delta\mu$ of the crisis with respect to the controlling parameter step $\delta\mu$ as following

$$\Delta\mu \propto \delta\mu/\lambda, \tag{6}$$

where λ is the maximal Lyapunov exponent.

The parameter shifts for both the boundary crisis and period-doubling bifurcations cause the so-called "artificial" *hysteresis* that the trajectory is still stay in a lose-stability attractor while another attractor has already existed.

Besides the enviroment parameter changing of of many dynamical systems, it should also pay attention on the method of extracting data from the natural systems, in which we usually use the average or sliding average values such as the economical data per hour, day, month and even year, and the b-values in seismic events which are the sliding averages of seismic events in a definite time interval[15,16]. The periodic orbits from these data are totally obscured. In Fig. 3 we shown the sliding average values of the SVS (2). It is clear that the periodic orbits correspond to some dips. This gives us a hint that a tangent bifurcation might happen at this parameter range and periodic orbits might exist.

Possible applications There are various applications of the developing diagram. Recently, short term predictability[17] on deterministic dynamical systems with fixed parameters has been obtained a great deal attention. Are the data from a dynamical system with changing parameter predictable? With our model, we can show that that *the short term predictability is possible for these natural phenomena no matter the environment parameter changes slowly or quickly*, provided that the changing of the parameter is governed by some rules (Though we might not know exactly what they are, see detail in our recent work, ref, 18). With this idea, we can understand that it is reasonable that some economical data[4] and the b-values[19] for seismic events are predictable.

The developing diagram can also be used to clarify the real physical meaning of the various characterizing quantities (correlation dimension, Lyapunov exponent, etc) which were calculated for many natural dynamical systems[18]. In fact, due to the changing of parameters, the reconstructed figures for the data from a developing diagram is very similar to that for a noised dynamical system.

An important application of the developing diagram lies in the study of the subsystems (or open systems) of dynamical systems. The influence of the other system can be considered as one or more changing parameters, which might be written by some equations phenomenally. This can help us to understand the restrictions of applying various perturbation and truncation theory to nonlinear systems since a small changing of the environment parameter(by adding high order perturbations or truncations) will suppress or induce chaos, crises[20].

Summary and discussion In summary, we have introduced a set of dynamical models with *parameter changing continuously and the accumulation of this change might be large*. The parameters for boundary crises and period-doubling bifurcations are shifted and scaling properties for these phenomena are exploited. Due to these shifts, the period-doubling bifurcations seem no longer continuous. We also discuss other possible applications of the models.

There are still a number of interesting questions remained to be investigated. Since the data series from our model (as well as many natural systems) are the set of transients for different parameters, how to extend the general methods[see, eg. 21,22] to extract information from the data series and then control and predict them is needed. The "artificial hysteresis" clearly has closely connection with many biological phenomena. The average values, also introduced in this Letter, should be paid more attention even for that from a dynamical systems with fixed parameters. We are afraid that a great part of noise we considered in

many natural systems results from the averaging process and the changing parameters. These questions and the other applications of the developing diagrams are now undertaken by the author and his collaborations. Some of them will be presented in an extended paper.

* Address for correspondence.

[1] C. Nicolis and G. Nicolis, Nature **311**, 529 (1984); *ibit* **329**, 523 (1987); P. Grassberger, *ibit* **323**, 609 (1986); *ibit* **329**, 524 (1987).

[2] P. Chen, System Dynamics Review, **4**, 605(1988).

[3] L.-Y. Cao *et al.*, submitted to Comput. Economics. In this paper, several Chinese macroeconomic data sequences, e.g., the national financial expenditure, total value of retain sales, gross output value of industry and monetary supply, are found short-term predictable.

[4] A. Garfinkel *et. al.*, Science **257**, 1230 (1992); Steven J. Schiff *et. al*, Nature **370**, 615 (1994).

[5] G.P. Pavlos *et al.*, Int. J. Bif. & Chaos, **4**, 87(1994).

[6] P. Grassberger and I. Procaccia, Physica **9**D, 189 (1983).

[7] Hao Bai-Lin, *Chaos* (World Scientific, Singapore, 1984).

[8] P. Cvitanovic, *Universality in Chaos* (Hilger, Bristol, 1984); F. C. Moon, *Chaotic and Fractal Dynamics* (Ichaca, New York, 1992).

[9] P. Pryant and K. Wiesenfeld, Phys. Rev. A **33**, 2525 (1986); S. T. Vohra, L. Fabiny, and K. Wiesenfeld, Phys. Rev. Lett. **72**, 1333(1994).

[10] N. Platt, E. A. Spiegel, and C. Tresser, Phys. Rev. Lett. **70**, 279 (1993); P. W. Hammer *et. al.*, Phys. Rev. Lett. **73**, 1095 (1994).

[11] Mingzhou Ding, E. Ott, and C. Grebogi, Phys. Rev. E **50**, 4228 (1994).

[12] C. Grebogi, E. Ott, and J.A. York, Phys. Rev. Lett. **48**, 1507 (1982); Phisica, **7**D, 181 (1983).

[13] R. B. May, Nature **261**, 459 (1976); L. P. Kadanoff, Physics Today (Dec.) **36**, 46 (1983).

[14] B.-L. Hao, Phys. Lett. A **86**, 267 (1981); M. Franszek and P. Pieranski, Canadian J. Phys., **63**, 488(1985).

[15] J. M. Carlson, J. S. Langer, and B. E. Shaw, Rev. mod. Phys. **53**, 657(1994).

[16] Q.-L. Li, L. Yu, B.-L. Hao, and J.-B. Chen, *Time and Space Scanning of the Frequency-Magnitude Relation for Earthquakes* (Chinese Earthquake Press, Beijing, 1979).

[17] J.D. Farmer and J.J. Sidorowich, Phys. Rev. Lett. **59**, 845(1987);M. Casdagli, Physica bf 35D, 335(1989); H.D.I. Abarbanel, R. Brown and J.B. Kadtke, Phys. Rev. A **41**, 1782(1990); J. Stark, J. Nonlinear Science, **3**, 197(1993).

[18] H.-P. Fang and L.-Y. Cao, Phys. Rev. E **51**, 6254 (1995)

[19] L.-Y. Cao, H.-P. Fang, Q.-L. Li, and J.-B. Chen, Int. J. Bif. & Chaos, accepted . In this paper, the *b*-value for the seismic events in Tangshan region in China is shown short-term predictable.

[20] H.-P. Fang, G.W. He and L.Y. Cao, Suppressing chaos and transient in parameter perturbed systems, Commun. Theor. Physics, accepted.

[21] N. Packard, J. crutchfield, J.D. Farmer, and R. Shaw, Phys. Rev. Lett. **45**, 712(1980).

[22] C. Grebogi, E. Ott, and J.A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).

Figure Captions

Fig. 1 The developing diagram of the logistic map in the parameter interval $\mu \in [1.6, 1.82]$ with a parameter step $\delta\mu = 10^{-5}$. The initial point x_0 at $\mu = 1.6$ is on the attractor.

Fig. 2 Same as Fig. 1 with $\mu \in [1.999, 2.003]$.

Fig. 3 The sliding average values of the Fig. 1 with 500 data a moving.